Bisimilarity, behavioral and logical equivalence for stochastic right coalgebras

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May 4, 2007

The relationship of bisimilarity, logical and behavioral equivalence to each other is well-researched for the interpretation of modal logics through stochastic Kripke models. It can be shown that two Kripke models are bisimilar (i.e. they are related through a span of morphisms) iff they are behavioral equivalent (i.e., they are related through a cospan), which in turn is equivalent to their being logical equivalent (i.e., given a state in one model, there exists a state in the other model satisfying the same formulas). This applies in particular to Markov transition systems. The underlying logic is in this case usually some kind of Hennessy-Milner logic with formulas $\top \mid \phi_1 \wedge \phi_2 \mid \langle a \rangle_q \phi$ for actions a and rational q, the intuition being that formula $\langle a \rangle_q \phi$ holds in state s iff the probability to hit a state that satisfies ϕ after action a is no smaller than q.

We generalize stochastic Kripke models and Markov transition systems to stochastic right coalgebras. These are coalgebras for a functor $\mathfrak{F} \circ \mathfrak{S}$, where $\mathfrak{F}$ is an endofunctor on the category of suitable measurable spaces, and $\mathfrak{S}$ is the subprobability functor that assigns a measurable space all its subprobabilities. The modal operators are generalized through predicate liftings which are set-valued natural transformations involving the functor. Formulas are then given through $\top \mid \phi_1 \wedge \phi_2 \mid \langle \lambda \rangle \phi$ with λ a predicate lifting. A state s satisfies formula $\langle \lambda \rangle \phi$ iff the coalgebra's dynamics moves it through λ into a state in which ϕ is satisfied. This logic is the probabilistic counterpart to coalgebraic logic as investigated, e.g., by L. Moss, D. Pattinson or L. Schröder in different categorical settings for set based functors.

Consider two states as equivalent iff they cannot be separated by a formula, so iff they have the same theory. This equivalence relation is investigated quite closely; most interesting is the observation that the factor spaces of coalgebras that are logical equivalent are Borel isomorphic. This enables constructing a cospan for logical equivalent coalgebras under a separation condition for the set of predicate liftings, which in turn entails that behavioral and logical equivalence are really the same. It is interesting to see that the functor's properties enter only through the separation of the set of predicate liftings, and that no additional restrictions on the functor are required.

We construct a span of morphisms from a cospan. The central argument here is a selection argument which gives us the dynamics of a mediating coalgebra from the domains of the cospan. This construction is used to establish that behavioral equivalent coalgebras are bisimilar, yielding the equivalence of all three characterizations of a coalgebra's behavior as in the case of Kripke models or Markov transition systems.

Actually, we consider a logic which is slightly more general than the kernel logic above. It incorporates arbitrary Boolean operators as well as infinitesimal operators like the largest and the smallest fixed point. This added generality comes from incorporating suitable natural transformations. It turns out that the proofs require only slightly more effort than those for the kernel logic.

*Research funded in part by *Deutsche Forschungsgemeinschaft*, grant DO 263/8-3, *Algebraische Eigenschaften stochastischer Relationen*