Uniform domain representations of ℓ^p-spaces

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Abstract

We consider separable, algebraic domain representations of topological spaces. Separable metric spaces admit such representations, and in [1] a method for giving effective domain representations of effective metric spaces was presented.

In this talk, which is based on [2], we focus on the complete, separable metric space ℓ^p of p-convergent sequences over $\mathbb{R}$, where p is a real parameter ≥ 1. In the context of domain interpretation of type theory, the notion of parametrization of domains has arisen (cf.[3]), formalising the concept of a family of domains depending uniformly on a parameter from a fixed domain. We give a uniform domain representation of each ℓ^p, similar to the construction in [1], and establish a link between the formal notion of a parametrization of domain theory and the manner in which ℓ^p depends on the parameter p. The possibility of using dependent type constructors over a parametrization, motivates these investigations.

Using domain representations, we inherit a computability theory on the represented topological space from domain theory. In this particular case, we arrive at uniform notions of computability on the ℓ^p-spaces. Moreover, continuous functions can be represented as elements of the function space of domains, and this fact has some interesting implications for the Banach space structure of ℓ^p. Although this is a case study of ℓ^p, most of the results have immediate generalisations.

References

[1] J.Blanck, Domain Representability of Metric Spaces. Annals of Pure and Applied Logic **83**, 225-247, North-Holland 1997.

[2] P.Køber, Uniform domain representations of ℓ^p-spaces. Mathematical Logic Quarterly **53**, 180-205, Wiley 2007.

[3] E.Palmgren and V.Stoltenberg-Hansen, Domain Interpretations of Martin-Löfs Partial Type Theory. Annals of Pure and Applied Logic **48**, 135-196, North-Holland 1990.