

# The Uniformity Principle for $\Sigma$ -definability with Applications to Computable Analysis

Margarita Korovina<sup>1</sup> and Oleg Kudinov<sup>2</sup>

<sup>1</sup> Universität Siegen, Germany, and IIS SB RAS, Novosibirsk, Russia  
korovina@brics.dk,

<sup>2</sup> Sobolev Institute of Mathematics,  
pr. Koptuga 4, 630090, Novosibirsk, Russia  
kud@math.nsc.ru

This work is the next step in a series of papers [3, 4] using arguments from definability theory to logically characterise computable continuous data. In order to do this we have proposed the notion of majorant-computability and developed logical approach to computability over continuous data. This approach is based on representations of continuous data by suitable structures without the equality test and  $\Sigma$ -definability in extensions of the structures by hereditarily finite sets. One of the main features of the notion of majorant-computability is that on the one side it is independent from concrete representations of the elements of structures on the other side it is flexible, i.e. we can change the language of  $\Sigma$ -formulas to express appropriate computability properties.

In this talk we introduce and study the language of  $\Sigma_K$ -formulas which is an extension of the language of  $\Sigma$ -formulas. This language simplifies reasoning about computability of higher type continuous data, and admits elimination of universal quantifiers bounded by computable compact sets. In order to show these properties we prove Uniformity principle for  $\Sigma$ -definability over the real numbers extended by open sets. Based on this principle we show that if we have a  $\Sigma_K$ -formula, i.e. a formula with quantifier alternations where universal quantifiers are bounded by computable compact sets, then we can eliminate all universal quantifiers obtaining a  $\Sigma$ -formula equivalent to the initial one. We illustrate how the language of  $\Sigma_K$ -formulas and Uniformity principle can be employed for reasoning about computability over continuous data.

## References

1. J. Barwise. *Admissible sets and Structures*. Springer Verlag, Berlin, 1975.
2. Yu. L. Ershov. *Definability and computability*. Plenum, New-York, 1996.
3. Margarita V. Korovina. Computational aspects of sigma-definability over the real numbers without the equality test. In Matthias Baaz and Johann A. Makowsky, editors, *CSL*, volume 2803 of *Lecture Notes in Computer Science*, pages 330–344. Springer, 2003.
4. Margarita V. Korovina and Oleg V. Kudinov. Towards computability of higher type continuous data. In S. Barry Cooper, Benedikt Löwe, and Leen Torenvliet, editors, *CiE*, volume 3526 of *Lecture Notes in Computer Science*, pages 235–241. Springer, 2005.
5. Klaus Weihrauch. *Computable Analysis*. Springer Verlag, Berlin, 2000.