

A synthetic account of Sequential Domain Theory

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Synthetic Domain Theory (SDT) was originally suggested by Dana Scott to obtain a uniform account of domain theory. In SDT the domain structure is intrinsic to a chosen class of sets with enough properties. There are different variations: complete Σ -spaces (Phoa), well complete objects (Longley & Simpson) or replete ones (Hyland & Taylor) which may lead to different classes of domains depending on the axiomatic setting.

Several flavours of domains have been described following that approach usually relying on realizability toposes as ambient category of sets, including sequential or stable domains [5, 3]. In *loc.cit.* Longley and van Oosten characterized Ehrhard’s dI-domains with coherence [1]. They used realizability over sequential algorithms, whereas in this talk we investigate realizability over *observably* sequential algorithms.

More precisely, we investigate to what extent the Locally Boolean domain $U \equiv [N \rightarrow N]$ (with N the bilifting of the natural numbers) can be used as a partial combinatory algebra to obtain a realizability model for “Sequential” Synthetic Domain Theory. The cartesian closed category of Locally Boolean domains with bistable functions [2] is interesting as it is equivalent to the category of sequential algorithms on sequential data structures of Lamarche and Curien.

Following ideas of [3] we give a model of SDT using modest sets on U where the intrinsic specialization (Σ -) order is the bistable order. There will be some canonical changes in the development of the SDT machinery: for instance bilifting replaces (normal) lifting and the isomorphism established by the **catch** operation $\Sigma^{\Sigma \times \Sigma} \cong \Sigma_{\perp}^{\top}$ replaces Phoa’s axiom.

The axiomatization of the model obtained, be it categorical or type theoretical (see e.g. [6, 4]), is rather challenging.

References

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