On Effectivity and Continuity of Multifunctions

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Extended abstract

Multifunctions have been used with great success in analysis and logic. We will consider such maps in the framework of second-countable T_0 spaces $\mathcal{T}$. In such spaces every point is uniquely determined by the collection of all basic open sets containing it, i.e. by a base of its neighbourhood filter.

We think of the basic open sets as easy to encode observations that can be made about the computational processes determining the elements. By doing better and better observations we will finally be able to determine every element. Thus, we assume that $\mathcal{T}$ not only comes with a fixed basis $\mathcal{B} = \{B_0, B_1, \ldots\}$, but also with a relation of definite refinement between the basic open sets which in many cases will be stronger than set inclusion. As it turns out in these cases, the refinement relation is a relation between the codes of the basic open sets and rather than the sets itself. In most applications it will be enumerable.

Therefore, we suppose that the indexing B of the basic open sets is such that there is a transitive recursively enumerable relation on the indices so that the property of being a topological basis holds with respect to this relation instead of just set inclusion. This leads us to the notion of an *effective space*. Note that we think of the topological basis with its numbering and the associated refinement relation as being part of the structure under consideration.

In this talk we will only be interested in *constructive* points. For any such point the collection of its elementary properties (basic open sets containing it) is enumerable.

A function between effective spaces that comes with a computable operation translating programs enumerating the elementary properties of the function arguments into programs enumerating the elementary properties of the function values is called *effective*. An important question is when such functions are effectively continuous.

In the talk we will deal with this question in the case of multifunctions. Note that various continuity notions for multifunctions have been discussed in the literature so far. We will introduce corresponding effective versions.

Multifunctions can be considered as set-valued maps. Therefore, in order to be able to talk about effective multifunctions we need to discuss what are suitable elementary properties of sets, how they can be coded, and which sets are determined by them. This will lead to different effectivity notions for multifunctions.

Finally, a general result on the effective continuity of effective multifunctions will be presented and some important special cases will be considered.